Exploration of Rational Exponents Name_____

Using your calculator, complete the following table.

Expression	Numerical Value	Expression	Numerical Value
$(4)^{\frac{1}{2}} = ?$	2	$\sqrt[2]{4^1} = \sqrt{4} = ?$	2
$(64)^{\frac{1}{3}} = ?$	4	$\sqrt[3]{64^1} = ?$	4
$(8)^{\frac{2}{3}} = ?$	4	$\sqrt[3]{8^2} = ?$	4
$(16)^{\frac{1}{4}} = ?$	2	$\sqrt[4]{16^1} = ?$	2
$(25)^{-\frac{1}{2}} = ?$	$\frac{1}{5}$	$\left(\sqrt[2]{25}\right)^{-1} = \frac{1}{\sqrt{25}} = ?$	$\frac{1}{5}$
$\left(2^3\right)^{\frac{1}{2}} = ? \approx$	2.828	$\sqrt[2]{(2)^3} = ? \approx$	2.828

1. What did you notice about your answers to the problems in the same rows?

The answers are the same.

2. Is there some pattern that relates the two expressions in each row to one another? Describe the pattern.

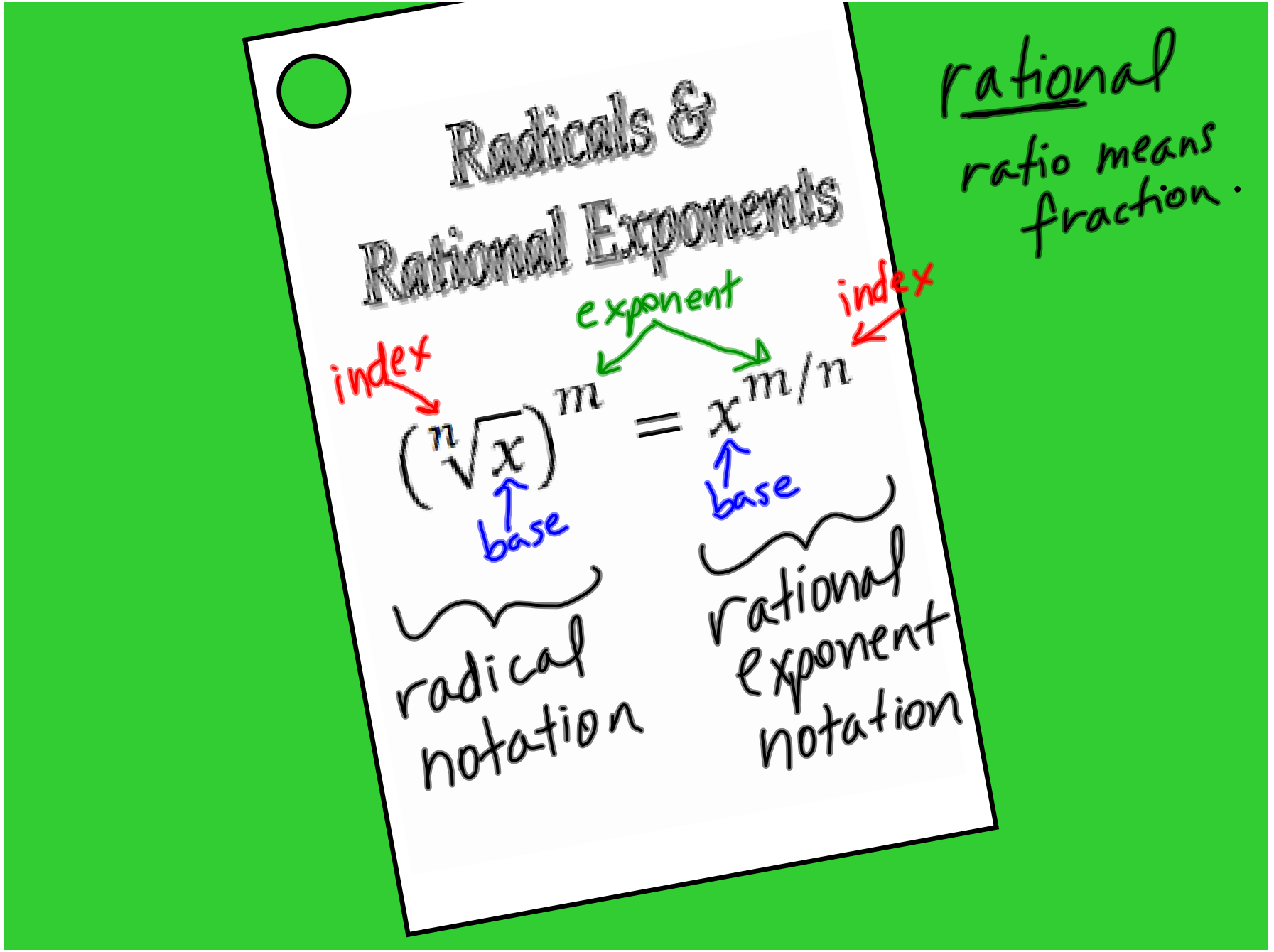
The denominator is the index of the root and the numerator is the exponent on the radical.

3. Given the expression $\left(5^{3}\right)^{\frac{1}{4}}$, what expression using a root symbol would yield the same numerical value?

$\left(5^{\frac{3}{1}}\right)^{\frac{1}{4}} = 5^{\frac{3}{4}} = \sqrt[4]{5^{3}}$

4. Given the expression $\sqrt[3]{54}$, what expression utilizing a fractional exponent would yield the same numerical value?

$\sqrt[3]{54^{1}} = 54^{1/3}$


Radicals &
Rational Exponents
index
exponent
index
$\left(\sqrt[n]{x}\right)^m = x^{m/n}$
base
base
radical notation
rational exponent notation
rational
ratio means fraction.

Example 1

Change between radical notation and rational exponent notation

Rewrite the expression using rational exponent notation.

1. $(\sqrt[5]{63})^3$

$63^{3/5}$

2. $(\sqrt[3]{-25})^4$

$(-25)^{4/3}$

3. $(\sqrt[6]{124})^7$

$124^{7/6}$

Rewrite the expression using radical notation.

4. $(-57)^{4/3}$

$\sqrt[3]{-57}^{4}$

5. $13^{3/2}$

$\sqrt{13}^{3}$

6. $204^{5/8}$

$\sqrt[8]{204}^{5}$

Checkpoint Complete the following exercises.

Rewrite using rational exponents.

a) $\sqrt{7} = 7^{1/2}$

b) $\sqrt[3]{5} = 5^{1/3}$

c) $\sqrt[5]{11} = 11^{1/5}$

Rewrite using radical notation.

d) $3^{1/2} = \sqrt{3}$

e) $14^{1/6} = \sqrt[6]{14}$

f) $4^{1/3} = \sqrt[3]{4}$

Example 2 Evaluate an expression without a calculator

a) $36^{3/2}$

$(6^2)^{3/2}$

$6^{\frac{2}{1}\cdot\frac{3}{2}} = 6^3 = 216$

b) $64^{-1/6}$

$(2^6)^{-1/6}$

$2^{\frac{6}{1}\cdot\frac{-1}{6}} = 2^{-1} = \frac{1}{2}$

c) $16^{-1/4}$

$(2^4)^{-1/4}$

$2^{\frac{4}{1}\cdot\frac{-1}{4}} = 2^{-1} = \frac{1}{2}$

d) $8^{4/3}$

$(2^3)^{4/3}$

$2^{\frac{3}{1}\cdot\frac{4}{3}} = 2^4 = 16$

Checkpoint Complete the following exercises.

Evaluate an expression without a calculator

a) $\sqrt[3]{8} = 8^{1/3} = (2^3)^{1/3} = 2$

b) $\sqrt[6]{64} = 64^{1/6} = (2^6)^{1/6} = 2$

c) $(-27)^{1/3} = (-3^3)^{1/3} = -3$

Example 3 Evaluate an expression with a calculator

a) $(\sqrt[3]{23})^5$

$\sqrt[3]{23}^5$
186.0143262
$23^{5/3}$
186.0143262

≈ 186.01

b) $(\sqrt[5]{-124})^4$

$(\sqrt[5]{-124})^4$
47.28651939
$(-124)^{4/5}$
47.28651939

≈ 47.29

c) $(-1012)^{8/5}$

$(-1012)^{8/5}$
64311.52678
$\sqrt[5]{-1012}^8$
64311.52678

≈ 64311.53

d) $(-128)^{-2/5}$

$(-128)^{-2/5}$
.1435872944
$\sqrt[5]{-128}^{-2}$
.1435872944

≈ 0.14

Checkpoint Complete the following exercises.

Evaluate an expression with a calculator

a) $\sqrt[3]{21}$

$\sqrt[3]{21}$ 2.758924176

$21^{1/3}$ 2.758924176

≈ 2.76

b) $\sqrt[6]{36}$

$\sqrt[6]{36}$ 1.817120593

$36^{1/6}$ 1.817120593

≈ 1.82

c) $(-37)^{1/3}$

$(-37)^{1/3}$ -3.332221852

$\sqrt[3]{-37}$ -3.332221852

≈ -3.33